

**SAMPLE PAPER - 03 (2019-20)**

**SUBJECT: MATHEMATICS**  
**CLASS : XII**

**MAX. MARKS : 80**  
**DURATION : 3 HRS**

**General Instruction:**

- (i) All the questions are compulsory.  
 (ii) The question paper consists of **36** questions divided into 4 sections A, B, C, and D.  
 (iii) **Section A** comprises of **20** questions of **1** mark each. **Section B** comprises of **6** questions of **2** marks each. **Section C** comprises of **6** questions of **4** marks each. **Section D** comprises of **4** questions of **6** marks each.  
 (iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.  
 (v) Use of calculators is not permitted.

**SECTION – A**

Questions 1 to 20 carry 1 mark each.

1. The value of  $\cot(\sin^{-1}x)$  is

- (a)  $\frac{\sqrt{1+x^2}}{x}$                       (b)  $\frac{x}{\sqrt{1+x^2}}$                       (c)  $\frac{1}{x}$                       (d)  $\frac{\sqrt{1-x^2}}{x}$

2. The matrix  $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$  is a

- (a) square matrix (b) diagonal matrix (c) unit matrix (d) none

3. If  $A = [a_{ij}]$  is a  $2 \times 3$  matrix, such that  $a_{ij} = \frac{(-i+2j)^2}{5}$ , then  $a_{23}$  is

- (a)  $\frac{1}{5}$                       (b)  $\frac{2}{5}$                       (c)  $\frac{9}{5}$                       (d)  $\frac{16}{5}$

4. If  $\int \frac{x^3}{\sqrt{1+x^2}} dx = a(1+x^2)^{\frac{3}{2}} + b\sqrt{1+x^2} + C$ , then

- (a)  $a = \frac{1}{3}, b = 1$                       (b)  $a = \frac{-1}{3}, b = 1$                       (c)  $a = \frac{-1}{3}, b = -1$                       (d)  $a = \frac{1}{3}, b = -1$

5. If  $\theta$  is the angle between any two vectors  $\vec{a}$  and  $\vec{b}$ , then  $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$  when  $\theta$  is equal to

- (a)  $\frac{\pi}{3}$                       (b)  $\frac{\pi}{2}$                       (c)  $\frac{\pi}{4}$                       (d)  $\frac{\pi}{6}$

6. The sine of the angle between the straight line  $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$  and the plane  $2x - 2y + z = 5$  is

- (a)  $\frac{10}{6\sqrt{5}}$                       (b)  $\frac{4}{5\sqrt{2}}$                       (c)  $\frac{2\sqrt{3}}{5}$                       (d)  $\frac{\sqrt{2}}{10}$

7. Distance of the point  $(\alpha, \beta, \gamma)$  from y-axis is  
 (a)  $\beta$  (b)  $|\beta|$  (c)  $|\beta| + |\gamma|$  (d)  $\sqrt{\alpha^2 + \gamma^2}$
8. The reflection of the point  $(\alpha, \beta, \gamma)$  in the xy– plane is  
 (a)  $(\alpha, \beta, 0)$  (b)  $(0, 0, \gamma)$  (c)  $(-\alpha, -\beta, \gamma)$  (d)  $(\alpha, \beta, -\gamma)$
9. Let A and B be two events such that  $P(A) = 0.6$ ,  $P(B) = 0.2$ , and  $P(A | B) = 0.5$ . Then  $P(A' | B')$  equals  
 (a)  $\frac{1}{10}$  (b)  $\frac{1}{30}$  (c)  $\frac{3}{8}$  (d)  $\frac{6}{7}$

10. Let X be a discrete random variable. The probability distribution of X is given below:

X	30	10	-10
P(X)	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{2}$

Then E (X) is equal to

- (a) 6 (b) 4 (c) 3 (d) - 5
11. If  $xy = 9$ , then  $\frac{dy}{dx} =$  \_\_\_\_\_
12. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be the functions defined by  $f(x) = x^3 + 5$ . Then  $f^{-1}(x)$  is \_\_\_\_\_
13. The value of c in Mean value theorem for the function  $f(x) = x(x - 2)$ ,  $x \in [1, 2]$  is \_\_\_\_\_
14. The value of m, for which the function  $f(x) = mx + c$  is decreasing for  $x \in \mathbb{R}$  is \_\_\_\_\_

**OR**

The rate of change of volume of a cone of constant height with respect to radius of the base is \_\_\_\_\_

15. If  $|\vec{a}| = \sqrt{3}$ ,  $|\vec{b}| = 2$  and angle between  $\vec{a}$  and  $\vec{b}$  is  $60^\circ$ , then  $\vec{a} \cdot \vec{b} =$  \_\_\_\_\_

**OR**

The projection of  $\vec{a}$  on  $\vec{b}$  if  $\vec{a} \cdot \vec{b} = 8$  and  $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$  is \_\_\_\_\_

16. Evaluate:  $\int_0^1 \frac{dx}{1+x^2}$

17. Find  $\int \frac{\cos 2x}{\cos x} dx$ .

18. Find the general solution of the differential equation  $e^x dy + (y e^x + 2x) dx = 0$ .

19. Find the value of the determinant  $\Delta = \begin{vmatrix} \sqrt{37} + \sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{15} + \sqrt{74} & 5 & \sqrt{10} \\ 3 + \sqrt{185} & \sqrt{15} & 5 \end{vmatrix}$ .

20. Find  $\int \frac{2}{1 + \cos 2x} dx$

**OR**

Find  $\int (e^{2x} + 5x)^2 (2e^{2x} + 5) dx$

**SECTION – B****Questions 21 to 26 carry 2 marks each.**

21. Let  $f : \{1, 3, 4\} \rightarrow \{1, 2, 5\}$  and  $g : \{1, 2, 5\} \rightarrow \{1, 3\}$  be given by  $f = \{(1, 2), (3, 5), (4, 1)\}$  and  $g = \{(1, 3), (2, 3), (5, 1)\}$ . Write down  $\text{gof}$ .

**OR**

Express  $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$ ,  $-\frac{\pi}{2} < x < \frac{\pi}{2}$  in simplest form.

22. Sand is pouring from a pipe at the rate of  $12 \text{ cm}^3/\text{s}$ . The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

23. Show that  $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$ , if  $\vec{a}$  and  $\vec{b}$  are along adjacent sides of a rectangle.

**OR**

If vectors  $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j}$  are such that  $\vec{a} + \lambda\vec{b}$  is perpendicular to  $\vec{c}$  then find the value of  $\lambda$ .

24. Show that the line through the points (1, -1, 2) and (3, 4, -2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

25. Show that the differential equation of which  $y = 2(x^2 - 1) + ce^{-x^2}$  is a solution, is  $\frac{dy}{dx} + 2xy = 4x^3$ .

26. A die is thrown three times, if the first throw results in 4, then find the probability of getting 15 as a sum.

**SECTION – C****Questions 27 to 32 carry 4 marks each.**

27. Determine whether the relation R defined on the set R of all real numbers as  $R = \{(a, b) : a, b \in R \text{ and } a - b + \sqrt{3} \in S\}$ , where S is the set of all irrational numbers, is reflexive, symmetric and transitive.

28. If  $y = \sin(m \sin^{-1} x)$ , prove that  $(1 - x^2) y_2 - xy_1 + m^2 y = 0$ .

**OR**

If  $y = \log\left(\frac{x}{a + bx}\right)^x$ , prove that  $x^3 \frac{d^2 y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^2$ .

29. Evaluate:  $\int \frac{1}{\cos^4 x + \sin^4 x} dx$

30. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs. 50 per kg to produce food I and Rs. 70 per kg to produce food II. Find the minimum cost of such a mixture. Formulate the above as an LPP mathematically and then solve it.

31. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be atleast 4 successes.

**OR**

In a bulb factory, machines A, B and C manufacture 60%, 30% and 10% bulbs respectively. 1%, 2% and 3% of the bulbs produced respectively by A, B and C are found to be defective. A bulb is picked up at random from the product and is found to be defective. Find the probability that this bulb was produced by the machine A.

32. Solve the differential equation  $(x^2 - yx^2) dy + (y^2 + x^2y^2) dx = 0$ .

### SECTION – D

Questions 33 to 36 carry 6 marks each.

33. Using properties of determinants, prove that 
$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

**OR**

If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ , find  $A^{-1}$  and hence prove that  $A^2 - 4A - 5I = O$ .

34. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

**OR**

Show that the semi-vertical angle of a right circular cone of given total surface area and maximum volume is  $\sin^{-1} \frac{1}{3}$ .

35. Using the method of integration find the area of the region bounded by the lines  $3x - 2y + 1 = 0$ ,  $2x + 3y - 21 = 0$  and  $x - 5y + 9 = 0$ .

36. Find the equation of the plane passing through the point  $P(1, 1, 1)$  and containing the line  $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} - 5\hat{k})$ . Also, show that the plane contains the line  $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(\hat{i} - 2\hat{j} - 5\hat{k})$ .

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