SAMPLE PAPER – 02 (2019-20)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**
CLASS : XII **DURATION : 3 HRS**

General Instruction:
(i) All the questions are compulsory.
(ii) The question paper consists of **36** questions divided into 4 sections A, B, C, and D.
(iii) **Section A** comprises of **20 questions** of **1 mark** each. **Section B** comprises of **6 questions** of **2 marks** each. **Section C** comprises of **6 questions** of **4 marks** each. **Section D** comprises of **4 questions** of **6 marks** each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The value of $\sin^{-1}\left(\cos\left(\frac{43\pi}{5}\right)\right)$

(a) $\frac{3\pi}{5}$ (b) $\frac{-7\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $-\frac{\pi}{10}$

2. If $A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$, then

(a) only AB is defined (b) only BA is defined
(c) AB and BA both are defined (d) AB and BA both are not defined.

3. The matrix $A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ is a

(a) scalar matrix (b) diagonal matrix (c) unit matrix (d) square matrix

4. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b} \geq 0$ only when

(a) $0 < \theta < \frac{\pi}{2}$ (b) $0 \leq \theta \leq \frac{\pi}{2}$ (c) $0 < \theta < \pi$ (d) $0 \leq \theta \leq \pi$

5. P is a point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If x co-ordinate of P is 5, then its y co-ordinate is

(a) 2 (b) 1 (c) –1 (d) –2

6. If α, β, γ are the angles that a line makes with the positive direction of x, y, z axis, respectively, then the direction cosines of the line are.

(a) sin α, sin β, sin γ (b) cos α, cos β, cos γ
(c) tan α, tan β, tan γ (d) $\cos^2 \alpha, \cos^2 \beta, \cos^2 \gamma$

7. The distance of a point P (a, b, c) from x-axis is

(a) $\sqrt{a^2+c^2}$ (b) $\sqrt{a^2+b^2}$ (c) $\sqrt{b^2+c^2}$ (d) b^2+c^2

8. Let A and B be two events. If P (A) = 0.2, P (B) = 0.4, P (A $\cup$ B) = 0.6, then P (A | B) is equal to

(a) 0.8 (b) 0.5 (c) 0.3 (d) 0

9. If A and B are any two events such that P(A) + P(B) – P(A and B) = P(A), then

(a) P(B|A) = 1 (b) P(A|B) = 1

(c) P(B|A) = 0 (d) P(A|B) = 0

10. $\int e^x(\cos x - \sin x)dx$ is equal to

(a) $e^x \cos x + C$ (b) $e^x \sin x + C$ (c) $-e^x \cos x + C$ (d) $-e^x \sin x + C$

11. If f : R $\rightarrow$ R be defined by $f(x) = (3 - x^3)^{1/3}$, then fof (x) = _____

12. If $f(x) = \begin{cases} ax+1, & if \quad x \geq 1 \\ x+2, & if \quad x<1 \end{cases}$ is continuous, then 'a' should be equal to _____.

13. In applying one or more row operations while finding A^{-1} by elementary row operations, we obtain all zeros in one or more, then A^{-1} _____.

14. The point on the curve $y = x^2$ does the tangent make an angle of 45° with the x-axis is _____

OR

The slope of the tangent to the curve $x = 3t^2 + 1$, $y = t^3 - 1$ at x = 1 is _____

15. If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 5\hat{k}$, then a unit vector in the direction of $\vec{a} - \vec{b}$ is _____

OR

The angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ is _____

16. Find the value of $\begin{vmatrix} 0 & xyz & x-z \\ y-x & 0 & y-z \\ z-x & z-y & 0 \end{vmatrix}$

17. Find: $\int (ax+b)^3 dx$

18. Find $\int \frac{2\cos x}{3\sin^2 x} dx$

19. Evaluate: $\int_0^{\frac{\pi}{2}} \cos x . e^{\sin x} dx$

OR

If $\int_0^a \frac{1}{1+4x^2} dx = \frac{\pi}{8}$, then find the value of 'a'.

20. Find the general solution of the differential equation $\frac{ydx - xdy}{y} = 0$.

SECTION – B

Questions 21 to 26 carry 2 marks each.

21. Form the differential equation of the family of parabolas having vertex at the origin and axis along positive y-axis.

22. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

OR

If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ show that $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular to each other.

23. Find the equation of the tangent to the curve $x^2 + 3y - 3 = 0$, which is parallel to the line $y = 4x - 5$.

24. Find the foot of the perpendicular drawn from the point A(1, 0, 3) to the join of the points B(4, 7, 1) and C(3, 5, 3).

25. Find the value of $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

OR

Let R be the relation in the set Z of integers given by R = {(a, b) : 2 divides a – b}.Show that the relation R transitive? Write the equivalence class [0].

26. A die is thrown three times, if the first throw results in 4, then find the probability of getting 15 as a sum.

SECTION – C

Questions 27 to 32 carry 4 marks each.

27. Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2+1}$, $\forall x \in R$ is neither one-one nor onto.

28. If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 y_2 + 2x (x^2 + 1) y_1 = 2$

OR

Differentiate the given function with respect to x : $y = \tan^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$

29. A diet for a sick person must contain at least 4,000 units of vitamins, 50 units of minerals and 1,400 calories. Two foods X and Y are available at a cost of 4 and 3 per unit respectively. 1 unit of the food X contains 200 units of vitamins, 1 unit of minerals and 40 calories, whereas 1 unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of X and Y should be used to have least cost, satisfying the requirements?

30. Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X. Also, find mean of the distribution.

OR

In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4 and 1 per cent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

31. Evaluate $\int \frac{dx}{\sqrt{5-4x-2x^2}}$

32. Solve the following differential equation $(1+e^{x/y})dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$.

SECTION – D

Questions 33 to 36 carry 6 marks each.

33. Draw the rough sketch of the region $\{(x, y) : y^2 \le 3x, 3x^2 + 3y^2 \le 16\}$ and find the area of the region enclosed by using the method of integration.

34. Using properties of determinants, prove that

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)(ax^2+2bxy+cy^2)$$

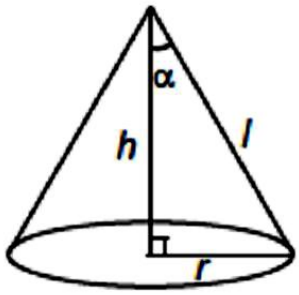
OR

If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

35. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

OR

Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1}\sqrt{2}$.



36. Find the equation of the plane passing through the intersection of planes $4x - y + z = 10$ and $x + y - z = 4$ and parallel to the line with direction ratios, 2, 1, 1. Find the perpendicular distance of the point (1, 1, 1) from this plane.