

SUBJECT: MATHEMATICS
CLASS : XII

MAX. MARKS : 80
DURATION : 3 HRS

General Instruction:

- (i) All the questions are compulsory.
- (ii) The question paper consists of **36** questions divided into 4 sections A, B, C, and D.
- (iii) **Section A** comprises of **20** questions of **1** mark each. **Section B** comprises of **6** questions of **2** marks each. **Section C** comprises of **6** questions of **4** marks each. **Section D** comprises of **4** questions of **6** marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted.

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The projection of vector $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ along $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ is
 (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\sqrt{6}$ (d) 2

2. $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to
 (a) $\tan x + \cot x + C$ (b) $(\tan x + \cot x)^2 + C$ (c) $\tan x - \cot x + C$ (d) $(\tan x - \cot x)^2 + C$

3. The coordinates of the foot of the perpendicular drawn from the point (2, 5, 7) on the x-axis are given by
 (a) (2, 0, 0) (b) (0, 5, 0) (c) (0, 0, 7) (d) (0, 5, 7)

4. The equations of x-axis in space are
 (a) $x = 0, y = 0$ (b) $x = 0, z = 0$ (c) $x = 0$ (d) $y = 0, z = 0$

5. A line makes equal angles with co-ordinate axis. Direction cosines of this line are
 (a) $\pm(1,1,1)$ (b) $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (c) $\pm\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$ (d) $\pm\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

6. If $P(A|B) > P(A)$, then which of the following is correct :
 (a) $P(B|A) < P(B)$ (b) $P(A \cap B) < P(A) \cdot P(B)$
 (c) $P(B|A) > P(B)$ (d) $P(B|A) = P(B)$

7. The value of $\tan\left(\cos^{-1} \frac{3}{5} + \tan^{-1} \frac{1}{4}\right)$ is
 (a) $\frac{19}{8}$ (b) $\frac{8}{19}$ (c) $\frac{19}{12}$ (d) $\frac{3}{4}$

8. Total number of possible matrices of order 3×3 with each entry 2 or 0 is
 (a) 9 (b) 27 (c) 81 (d) 512

9. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

(a) $\frac{1}{10}$ (b) $\left(\frac{1}{2}\right)^5$ (c) $\left(\frac{9}{10}\right)^5$ (d) $\frac{9}{10}$

10. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of $x + y$ is

(a) $x = 3, y = 1$ (b) $x = 2, y = 3$ (c) $x = 2, y = 4$ (d) $x = 3, y = 3$

11. If $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ are given by $f(x) = \sin x$ and $g(x) = 5x^2$, then $\text{gof}(x) =$ _____

12. If A and B are matrices of same order, then $(3A - 2B)'$ is equal to _____.

13. The point on the curve $x^2 + y = 4$, tangent is parallel to the x-axis is _____

OR

For the function $y = x^3$, if $x = 5$ and $\Delta x = 0.01$, then $\Delta y =$ _____

14. If function $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{if } x \neq 1 \\ k, & \text{if } x = 1 \end{cases}$ is continuous at $x = 1$, then the value of k is _____

15. If $\cos 2\theta = 0$, then find the value of $\begin{vmatrix} 0 & \cos \theta & \sin \theta \\ \cos \theta & \sin \theta & 0 \\ \sin \theta & 0 & \cos \theta \end{vmatrix}^2$.

16. Find the general solution of a differential equation of the type $\frac{dx}{dy} + Px = Q$.

17. If $\int (ax+b)^2 dx = f(x) + C$, then find $f(x)$.

18. Find: $\int \frac{1}{(3+x)^2+1} dx$

19. Find $\int (e^x + 3x)^2 (e^x + 3) dx$

OR

Find $\int \frac{2}{1+\cos 2x} dx$

20. The unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$ is _____

OR

If $\vec{AB} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{BC} = 6\hat{i} + 3\hat{j} - 6\hat{k}$, then the points A, B, C are _____

SECTION – B

Questions 21 to 26 carry 2 marks each.

21. Let $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$, then find the value(s) of α for which $f \circ f$ is identity function
 $\alpha \in \{\sqrt{2}, -\sqrt{2}, 1, -1\}$.

OR

Express $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$ in simplest form where $-\frac{\pi}{4} < x < \frac{\pi}{4}$.

22. Form the differential equation representing the family of curves $y = e^{2x} (A + Bx)$, where A and B are constants.
23. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

OR

If two vectors \vec{a} and \vec{b} are such that $|\vec{a}|=2, |\vec{b}|=1$ and $\vec{a} \cdot \vec{b} = 1$, then find the value of $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$.

24. The x-coordinate of a point P on the line joining the points Q(2, 2, 1) and R(5, 1, -2) is 4. Find its z-coordinate.
25. Find the interval in which the function $f(x) = 2x^3 - 15x^2 + 36x + 17$ is strictly increasing or strictly decreasing.
26. If $P(A) = 0.4$, $P(B) = p$ and $P(A \cup B) = 0.7$. Find the value of p, if A and B are independent events.

SECTION – C

Questions 27 to 32 carry 4 marks each.

27. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that $f \circ f(x) = x$ for all. What is the inverse of f?

28. Differentiate $x^{x \cos x} + \frac{x^2+1}{x^2-1}$ w.r.t. x

OR

If $x = \sqrt{a^{\sin^{-1}t}}$, $x = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$.

29. In an examination, an examinee either guesses or copies or knows the answer of multiple choice questions with four choices. The probability that he makes a guess is $\frac{1}{3}$ and probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered it.

OR

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

30. Find the following integral $\int \frac{x^2+1}{x^2-5x+6} dx$.
31. Find the particular solution of the differential equations $(3xy + y^2)dx + (x^2 + xy)dy = 0$: for $x = 1, y = 1$.

32. A company manufactures three kinds of calculators: A, B and C in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind A, 4000 of kind B and 4800 of kind C. The daily output of factory I is of 50 calculators of kind A, 50 calculators of kind B, and 30 calculators of kind C. The daily output of factory II is of 40 calculators of kind A, 20 of kind B and 40 of kind C. The cost per day to run factory I is Rs. 12,000 and of factory II is Rs. 15,000. How many days do the two factories have to be in operation to produce the order with the minimum cost? Formulate this problem as an LPP and solve it graphically.

SECTION – D

Questions 33 to 36 carry 6 marks each.

33. Prove without expanding that
$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right).$$

OR

Solve the system of the following equations: $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$; $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

34. Show that the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$.

OR

An open topped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

35. Using integration, find the area of the region bounded by the triangle whose vertices are $(-1, 2)$, $(1, 5)$ and $(3, 4)$.
36. Find the equation of the plane passing through the points $(-1, 2, 0)$, $(2, 2, -1)$ and parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.